Fluid Dynamics: Examples

< Potential flow>

1. Consider a two-dimentional steady state potential flow of an inviscid incompresiive fluid, whose complex potential is given by

$$W = U_0(z + \frac{a^2}{z}).$$

Here, $z$ is the comlex, which is expressed by

$$z = x + iy = re^{i\theta},$$

and $a$ is a possitive constant. Answer the following questions.

(1) Calculate the real and imaginary parts of $W$, and then obtain the velocity potential $\phi(r,\theta)$ and stream function $\varphi(r,\theta)$, respectively.

(2) Using the result of question (1), obtain the radial velocity componet $u_r(r,\theta)$ and circumferential velocity component $u_\theta(r,\theta)$.

(3) Using the result of question (2), obtain the velocity componet in the $x$–direciton $u_x(r,\theta)$ and elocity componet in the $y$–direciton, $u_y(r,\theta)$.

(4) Using the result of question (2), explian the flow field at $r = a$.

(5) Using the result of question (3), explian the flow field far from the origin. $r = a$.

(6) Draw the flow field given by the potentail, $W$.

2. Consider the following complex velocity potential, $W$, which is given by

$$W = \alpha \ln z.$$

Here, $m$ and $z$ are a complex constant and variable, respectively, given by

$$\alpha = a - ib \quad (a \geq 0, b \geq 0),$$
$$z = x + iy = re^{i\theta},$$

And the following equations give $Q$ and $\Gamma$;

$$Q = \oint_C (\boldsymbol{u} \cdot \boldsymbol{n}) ds,$$

$$\Gamma = \oint_C \boldsymbol{u} \cdot d\boldsymbol{s},$$

where $C$ is a closed loop oriented counterclockwise around the origin. Answer the following questions.

(1) Calculate the real and imaginary parts of $W$, and then obtain the velocity potential $\phi(r,\theta)$ and stream function $\psi(r,\theta)$.

(2) Using the result of question (1), obtain the radial velocity component $u_r(r,\theta)$ and the circumferential velocity component $u_\theta(r,\theta)$.

(3) Draw the streamlines and evaluate $Q$ and $\Gamma$ in case of $b = 0$.

(4) Draw the streamlines and evaluate $Q$ and $\Gamma$ in case of $a = 0$.

3. Consider a two-dimensional steady state potential flow of an inviscid incompressible fluid around a corner as shown in Fig.1, whose complex velocity potential, $W$, is given by

$$W = A\,z^{\alpha}.$$

Here, $A$ and $z$ are the complex constant and variable, respectively, given by

$$A = |A| e^{i\beta} \quad , \quad z = x + iy = re^{i\theta} \quad ,$$

and $\alpha$ is a positive constant and $\beta$ is a constant of $-\pi < \beta < \pi$. Answer the following questions.

(1) Calculate the real and imaginary parts of $W$, and then obtain the velocity potential $\phi(r,\theta)$ and stream function $\psi(r,\theta)$.

(2) Using the result of question (1), obtain the radial velocity component $u_r(r,\theta)$ and the circumferential velocity component $u_\theta(r,\theta)$. Then obtain the velocity component in the $x$-direction, $u_x(x,y)$ and the velocity component in the $y$-direction, $u_y(x,y)$.

(3) Using the result of question (2), determine $\beta$ from a boundary condition in terms of $u_\theta(r,0)$.

(4) Using the results of questions (2) and (3), determine $\alpha$ from a boundary condition in terms of $u_\theta(r,\frac{3}{2}\pi)$.

(5) Using the results of questions (2), (3) and (4), obtain the absolute value of the flow velocity at the origin.

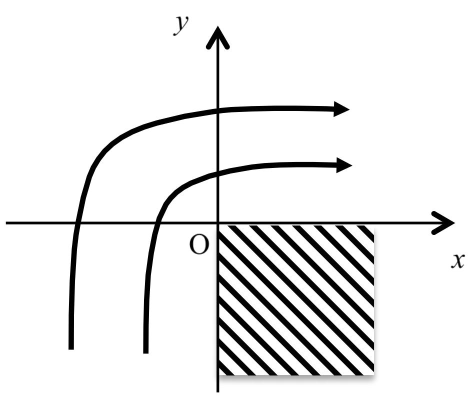


Fig.1

<Channel flow>

1. As shown in Fig. 2, the two types of fluid A and B flow in the $x$ direction between the parallel plates with a constant pressure gradient, $\frac{dp}{dx} = F_0\ (<0)$ without mixing with each other. It is assumed that the flow is sufficiently developed, and the width for the fluids A flow is "$a$" and that for the fluid B is "$b$". In addition, the viscosity coefficients of fluid A and fluid B are given by $\mu_A$ and $\mu_B$, respectively. And it is also assumed that the adhesive condition is satisfied on the wall. Answer the following questions.

(1) Show the velocity conditions on the wall.
(2) Show the conditions that hold at the boundary between fluids A and B.
(3) Find $U(y)$.
(4) Using the results of Question (3), show a schematic diagram of the flow velocity distribution in the case of $\mu_A > \mu_B$.

(4) Write down what kind of phenomenon will occur when $F_0\ (<0)$ becomes smaller from this state.

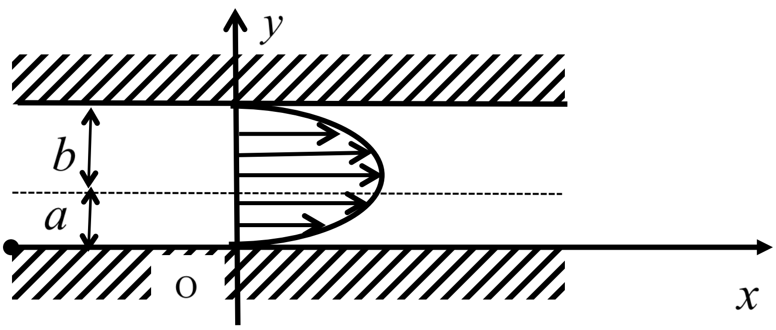


Fig. 2

2. Consider a concentric double circular pipe with an inner radius "$a$", an outer radius, "$b$ $(b > a)$"and a length unit, as shown in Fig. 3. The gap between the pipes is filled with a fluid having a viscosity coefficient, $\mu$, the outer pipe is mechanically fixed, and the inner pipe is rotated at an angular velocity, $\omega$. Here the flow is laminar and the adhesive condition is satisfied on the wall. Answer the following questions.

(1) Show the conditions that hold on the wall.
(2) Find the flow velocity distribution.
(3) Find the torque required to rotate the inner tube.
(4) Find the torque required to fix the outer pipe.

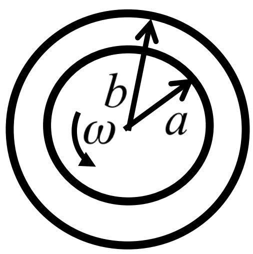


Fig.3